Optimal Control of Capital Injections by Reinsurance and Investments

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Abstract

We consider a classical continuous time risk model, where the claims are reinsured by some reinsurance with deductible $b \in [0, \tilde{b}]$, where $b = \tilde{b}$ means "no reinsurance" and $b = 0$ means "full reinsurance". In addition to that the insurer is allowed to invest into a risky asset modeled as a Black-Scholes Model or, if his surplus is positive, into a riskless asset with a fixed rate of interest $m > 0$. We choose as a risk measure, connected to some insurance portfolio, the value of expected discounted capital injections, which are necessary to keep the risk process above zero. Our goal is to find the value function defined as the infimum of expected discounted capital injections over all reinsurance and investment strategies and to derive the optimal strategy leading to the value function.

In the risky asset case we can not show the existence of a classical solution to the corresponding HJB equation and have to use the concept of viscosity solutions. In the case of the riskless interest rate we show the existence of a unique weak solution.

All considerations are illustrated with examples.

Key words: optimal control, stochastic control, Hamilton–Jacobi–Bellman equation, dividend, capital injections, classical risk model, viscosity solutions.

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